Determination of Longitude on the Planet Jupiter.
By G. W. Hough.

In the *Monthly Notices*, vol. lxiv. pp. 824–834, I published an article on longitude determinations on the planet *Jupiter.*

It would appear from a paper by Mr. A. S. Williams in *Monthly Notices*, vol. lxv. pp. 167–181, on the same subject that I failed to make clear some points in the discussion. Some further explanation may, therefore, be desirable.

The eye-estimate method has been used by many distinguished astronomers in the past in determining the rotation period of *Jupiter* and *Saturn*, and in common with everybody I imagined that it was a fairly reliable method. I had very little faith in Schmidt's variable error, and also supposed there was a *real* personal equation between different observers. The comparison of eye-estimates with the micrometer measures, however, has shown such grave errors that astronomers ought clearly to understand that a precision observation cannot be made by this method.

In meridian observations the transit of a star is observed over a group of fixed wires, from five to fifteen in number. Suppose all the wires were removed and the transit observed over an imaginary wire bisecting the field, the latter being analogous to that employed in eye-estimates. How would the two methods compare in point of accuracy in right ascension work?

The observations of the Barnard White Spot on *Saturn* in 1903, the only conspicuous spot on the disc, showed that experienced observers differed nearly twenty minutes on the same night, not once, but repeatedly. I think astronomers would not regard such as precision observations. I have already shown that a micrometer used for a fraction of the time required to secure such crude estimates would furnish observations of precision such as are demanded in other directions. If, therefore, anything I can say will induce observers who have micrometers to use them when we discover another suitable spot for determining the rotation of *Saturn* my time will not be entirely wasted.

In my previous paper I compared the micrometer results with a previously computed ephemeris by Marth to show that there was no variable or cumulative error.

As all the conclusions arrived at are based on the assumption that micrometer work is subject to accidental error only, some further explanation on this point may be desirable.

In the determination of longitude the central meridian of the disc is not directly used. The measures are referred to the limbs of the planet, and every measure for longitude is virtually a measure of the equatorial diameter, the spot or marking serving

as an intermediate step. In fact, the constants for the size of the disc were determined from such observations.

As the size of the disc from these differential measures is in harmony with direct measures it is obvious that there was no variable or cumulative error. When objects are observed at a considerable distance from the central meridian the accidental error, or the time of passage over the central meridian, may be materially increased owing to errors in the adopted constants for reduction—viz. the size of the disc, latitude, and length of the object.

If measures were always made near the central meridian I have good grounds for thinking that the mean accidental error would conform to theory—viz. one minute of rotation time.

In order to ascertain the mean personal equation and variable error all the observations used in the discussion were compared with an ephemeris derived from the micrometer measures.

For convenience in some cases the Marth-Crommelin ephemeris was corrected to conform to the true rotation-period; then both the micrometer and eye-estimates were compared, as in the example given for 1887. The residual errors, therefore, in all cases depend on the rotation-period derived from the micrometer measures.

In the eye-estimate method it seems to me there are two sources of error of vital importance.

First, personal equation.

Second, variable error, first pointed out by Schmidt.

Personal Equation.

What does personal equation indicate in eye-estimate observations? If the personal equation amounts to six minutes it means that the observer divided the disc into two *unequal parts*; that his central meridian, which he thought bisected the disc, was 1″ on one side, or one-fortieth of the diameter of the disc in error.

The range in the mean personal equation for different years (*Monthly Notices*, vol. lxiv. p. 831) is about eight minutes, which means that whenever the personal equation is changed a new central meridian is chosen. Schmidt found, from eye-estimates alone, a range of nine minutes in the personal equation for different observers. Now if the personal equation varies over such wide limits for the same observer, or for different observers, it means that the disc is not bisected by this method, and precision observations are not made. Hence, also, we may conclude that the range in personal equation is a correct measure of the amount of error in eye-estimates. It has also been shown that the personal equation is not the same for different spots observed on the same night, nor for the same spot at different oppositions.

Variable Personal Equation.

But the most serious error is the "variable personal equation," which is introduced when observations extend over a considerable interval of time. I think the term "variable personal equation" has been misunderstood; the designation "cumulative error" is preferable.

During an opposition spots or markings may be observed for 200 days or more. If, then, at the beginning of the series the micrometer and the eye-estimate are in agreement, and at the end of the series there is a difference of ten minutes more or less, and this difference varied substantially with the time, such difference has been designated as "variable personal equation" or cumulative error. It is not to be confounded with accidental error or constant error. In all the sets of observations which I compared with the micrometer, in which this error was apparent, with one exception, the difference between the micrometer and the observer increased or decreased with the time.

Variable personal equation, then, means that the observations are not referred to the same central meridian, but are gradually shifted to one side.

In this discussion nineteen sets of observations, made by five different persons, were compared with the micrometer, and in fifteen sets the cumulative error was well defined. That it may clearly be seen what effect "variable personal equation" has on the rotation period, I have determined it for the values given in *Monthly Notices*, vol. lxiv. p. 829.

			Time interval in Days.	Variable personal equation. min.	Error on rotation-period. sec.
Barnard	1891	Red Spot	122	5	1·0
,,		Long Red	85	0	0·0
,,		Small Black (a)	92	15	4·0
,,		,, (b)	85	7	2·1
Gledhill	1898	Black	144	10	1·7
Williams	1900	Red Spot	113	6	1·3

For the above six sets of observations only one, Barnard, Long Red, was free from "variable personal equation," and hence gave the correct rotation-period.

The "variable personal equation" or cumulative error may be shown to exist independently of the micrometer observations by simply comparing the rotation periods given by different observers for the same spot at the same opposition.

Schmidt's observations on the Red Spot (*Ast. Nach.* 2410) have been repeatedly quoted as a specimen of good eye-estimate work. The comparison with the micrometer measures may be of interest.

I have reduced the observations made between 1880 August 3

and 1881 March 20, a time interval of 229 days, with a variable increasing rotation-period, and compared them with the micrometer observations covering the same interval, with the following results :—

Mean personal equation	$-3{\cdot}04$ min. (99 obs.)
Variable personal equation ...	3
Mean O—E	$\pm 2{\cdot}65$
Mean actual error	$\pm 4{\cdot}05$

Schmidt found for a uniform rotation period O—E $\pm 3^{m}{\cdot}0$.

The four observations made in 1879 were not used, but were reduced separately. When these are included the variable personal equation is about 6 min.

Motion in Longitude.

The observations of the past twenty-five years have shown that the motion of the spots and markings seen on the planet *Jupiter* is smooth, never abrupt, as has been imagined by some observers. The rotation-period may be regarded as constant for a short time interval, sometimes for the whole opposition ; but in some cases the variation is so great that the observations can only be satisfied by assuming a uniformly decreasing or increasing period.

If observations could be made with greater precision I imagine a variable rotation-period would be apparent in all cases. As the labour required to compute an ephemeris for a variable period is considerable, it is only used when the observations made during an opposition cannot be fairly satisfied with a uniform period. At mean distance 1″ represents 2300 miles, and hence a spot 1″ in diameter represents about 4,000,000 square miles.

The objects that are observed are usually many millions of miles in area, and presumably have mass. We should not expect any abrupt change in direction or rate of motion in a moving mass.

My observations of a White Spot made in 1881 have been quoted a number of times in proof of the abrupt displacement of a spot on the disc of the planet. I find three observations, October 8, 18, 20, and again three others, November 21, 24, 30, which show abnormal residuals (O—E) about 10 min. greater than the mean of the residuals for the whole opposition of 252 days. As such irregularities are not found in any other set of measures, we should not be warranted for an isolated case in assuming irregular motion, even if there was no way to account for it. But the explanation is simple. This spot, which was observed for a number of years, appeared for a short time in 1881 as a long rift in the equatorial belt, its general appearance being an oblong spot. The apparent displacement was simply

due to the use of a different reference point in making the measures. When the spot appeared as a long rift the preceding end was used, otherwise the middle of the spot.

When the rotation-period changes a number of seconds during an opposition one might naturally infer irregular motion if the observations are compared with an ephemeris based on a uniform period.

Many cases of alleged irregular motion may possibly be explained on this hypothesis.

An example of irregular motion may be found in *Ast. Nach.* 3354.

A spot observed for 205 days showed residuals $+12^m$ at the beginning and end of the series, and -3^m near the middle. When, however, the observations were compared with an ephemeris based on a uniformly increasing period they were well satisfied, and gave a mean residual $(O-E)+2^m{\cdot}0$.

Subsequent to 1879, the observations of the Red Spot covering three successive oppositions were fairly represented by assuming a uniformly increasing rotation-period. An examination of the residuals, however, indicated that the initial rotation-period was not quite correct, or that a third term was required fully to satisfy the observations.

In general, whenever there is a marked difference in the magnitude of the residuals at the ends and middle of a series it indicates a variable rotation-period.

The position in regard to eye-estimates may be summarised as follows :—

(1) The actual error of an observation is about four or five minutes on the average.

(2) The range of personal equation for different observers may reach nine minutes (Schmidt).

(3) The range of personal equation for the same observer in different years may reach eight minutes.

(4) The range in personal equation in a set of observations extending over a long time interval may exceed ten minutes.

(5) There is no *fixed* personal equation. Vide *Monthly Notices*, vol. lxiv. pp. 829–831. The term simply means the average "mean" error as referred to the true central meridian of the disc. The fictitious central meridian chosen for any group seems to be accidental, since for different spots observed on the same night the central meridian is different. To sum up the whole matter, we may say that for a space of 2″ on the centre of the disc an observer may choose a fictitious central meridian thinking it is correct.

(6) All observers usually introduce an error depending on the time, which I have designated "variable personal equation" or cumulative error. This error is so peculiar that it is worthy of serious study. Such an error is not found in any kind of observations in the astronomy of precision. It appears, however, to be common to all observers in eye-estimates. The nature of

this error will be apparent by reference to the table of longitude given by Mr. Williams in *Monthly Notices* (Red Spot, 1887), vol. lxv. p. 175. At this time the rotation-period was practically stationary, and a constant longitude was maintained when the observations were referred to Marth's ephemeris.

Schmidt thought the variable error depended on the hour-angle, which *indirectly* depends on the time.

How does this peculiar error originate? A possible explanation is that in some way subsequent observations are influenced by preceding ones. When the rotation-period conforms very closely to a previously prepared ephemeris, as has been the case for the Red Spot, the presupposed time has a very important bearing on the observations made.

For some years the Red Spot was for a large portion of the time invisible, and its position could only be determined by the hollow in the belt; an object not well adapted for an observation of precision. We find, however, eye-estimates giving an average mean residual $O - E = \pm 1{\cdot}0^m = 0''{\cdot}2$. Such a degree of precision is seldom reached with the micrometer in the observation of well-defined spots, and is far beyond anything possible for the micrometer in the observation of the hollow.

The eye-estimate method for ascertaining the rotation time of a planet is simple and direct, and will continue to furnish observations of value. When the time interval is long, viz. between two oppositions, the errors I have investigated, accidental, personal equation, and cumulative, will have but little effect on the mean rotation-period derived.

It would be a curious anomaly, however, in astronomical development if there could be no improvement on a method devised more than two centuries ago, when modern instruments of precision were unknown.

Not very long ago, before equatorial mountings and driving clocks were in common use, the ring micrometer was pretty generally used for fixing the place of a comet. At the present day, however, I think few astronomers would make use of it except as a last resort.

The modern micrometer enables the observer to measure by repetition spaces smaller than can be seen with the telescope.

The principles involved in measurement are the same, whether the instrument is used for ascertaining the distance of two stars, or the distance of markings on a planetary disc. It strikes me, therefore, as a self-evident proposition that one could bisect a planetary disc with greater precision by the help of a micrometer than without one.

The Equatorial and Polar Diameters of Jupiter as measured with the Greenwich Transit-Circle, 1880–1901. By A. M. W. Downing, D.Sc., F.R.S.

In meridian transit observations the diameter of a planet may be defined as the perpendicular distance between the verticals which are tangents to the first and second limbs of the planet. This may be called the horizontal diameter at meridian passage. In the same way the vertical diameter at meridian passage is the perpendicular distance between the horizontal tangents to the north and south limbs. In the case of *Jupiter* the horizontal and vertical diameters are not generally the same respectively as the equatorial and polar diameters owing to the inclination of the planet's axis to the circle of declination.

Let P = the position-angle of the planet's axis

$$c = \sqrt{1-e^2} = \frac{b}{a} = \frac{\text{polar diameter}}{\text{equatorial diameter}}$$

$$\cot \mathrm{P}' = \frac{\cot \mathrm{P}}{c}, \text{ and } \tan \mathrm{P}'' = \frac{\tan \mathrm{P}}{c}$$

then, with sufficient accuracy in the case of *Jupiter*,

$$\text{hor. diam.} = \text{equat. diam.} \times \frac{\cos \mathrm{P}}{\cos \mathrm{P}'}$$

and

$$\text{vert. diam.} = \text{polar diam.} \times \frac{\cos \mathrm{P}}{\cos \mathrm{P}''}$$

Also let the true diameter = tabular diam. $(1+y)$, and let z be the correction to the adopted value of $\frac{b}{a}$ or c; then the equations of condition furnished by the meridian observations of diameters are:

(1) From the transit observations

$$\text{equat. diam.} \times \frac{\cos \mathrm{P}}{\cos \mathrm{P}'} \times y + \text{equat. diam.} \times \sin \mathrm{P} \,.\, \sin \mathrm{P}' \times z$$

$$= \text{observed correction to tabular horizontal diameter.}$$

(2) From the Z.D. observations

$$\text{polar diam.} \times \frac{\cos \mathrm{P}}{\cos \mathrm{P}''} \times y - \text{polar diam.} \times \sin \mathrm{P} \,.\, \sin \mathrm{P}'' \times z$$

$$= \text{observed correction to tabular vertical diameter.}$$

The variation of its coefficient is not sufficiently great to enable us to determine z satisfactorily in this way, and it seems better to adopt the value of c corresponding to the compression